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Precise Lunar Gravity Assist Transfers to Geostationary Orbits

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Introduction

THE transfer to geostationary orbit (GSO) is usually achieved by placing the spacecraft initially in a geostationary transfer orbit (GTO) with perigee altitude around 200 km and apogee around 36,000 km. The GTO orbital planes are inclined to the Earth equator because of launch station locations. A large amount of propellant is required to effect the plane change to achieve zero inclination as well as to raise the perigee altitude to 36,000 km. These maneuvers make the mission expensive, especially when the initial GTO inclinations are high. Alternate approaches[1–3] that advocate the use of lunar gravity assist are discussed in the literature to reduce the fuel budget.

When a geocentric trajectory goes through the lunar gravity field, it undergoes a plane change and gains or loses energy relative to the Earth. This phenomenon can be judiciously used to raise the perigee of the return trajectory, rotate the apsidal line, and change the orbital inclination by choosing appropriate initial transfer orbit characteristics relative to the Earth. Thus, the transfer of a spacecraft to GSO from GTO involves identification of appropriate initial transfer trajectory characteristics that result in a low inclination and GSO altitude as its perigee altitude after encounter with the moon.

In this Note, a numerical search technique that uses genetic algorithms (GA) is formulated. Because of the extreme sensitivity of the outgoing trajectory to the initial conditions, the performance of the regular GA[4] is found to be inadequate. A modified version of GA, GA with adaptive bounds (GAAB),[5] has successfully been employed to overcome the problem of high sensitivity. In this approach, the parameter bounds of GA are modified during the search process. The adaptation process helps generate precise lunar gravity assist trajectory design. Furthermore, the influence of different propagation force models on the initial conditions and on the achieved target parameters is assessed. The significance of Earth's second zonal harmonic is established.

Problem Description

The transfer trajectory characteristics are described by six parameters: semimajor axis a, eccentricity e, inclination i, right ascension of ascending node Ω, argument of perigee ω, and true anomaly ν. All of these parameters must be obtained at the time of departure. From the consideration of minimum energy, the inclination and the perigee point of GTO and transfer trajectory are taken to be the same. This fixes the true anomaly to zero. Because the transfer trajectory plane contains the moon, an equation relating Ω and the position of the moon at the encounter time can be written as[6,7]

$$\sin(\alpha_M - \Omega) = \tan\delta_M / \tan i \tag{1}$$

where α_M and δ_M are right ascension and declination of the moon. Also, the requirement of near zero inclination for the return trajectory establishes the moon's presence near the node at the time of encounter with the moon. That is, $\delta_M = 0$. Thus, the feasible range for Ω is in the neighborhood of α_M. Solutions can exist for various Ω in the feasible region, which is equivalent to various approach flight durations. With fixed Ω in this fashion, the unknown parameters that must be determined reduce to two, h_a and ω, where h_a is the apogee altitude. The objective is to achieve GSO altitude and inclination for the return orbit perigee altitude and inclination after lunar encounter. The problem is solved using a numerical search technique that uses numerical integration for propagation and GA concepts for regulating the search.

Numerical Search Technique Using GA

Genetic algorithms require a range of values to choose from for each of the unknown parameters. Fixing the range of values in the neighborhood of appropriate values enables fast convergence. The bounds for the argument of perigee altitude are fixed based on the observation that the transfer angle is around 180 deg and the perigee of the transfer orbit must be near the Earth equator. There are two possible ranges for argument of perigee: 1) near 0 deg and 2) around 180 deg. The ranges for apogee are fixed in the neighborhood of geocentric radial distance of the moon at the encounter time.

Fitness Function

The fitness value for each set of input parameters is given by

$$f = 1/(1 + \mathrm{obj}) \tag{2}$$

where

$$\mathrm{obj} = [(h_f - h_{\mathrm{GSO}})/w_h]^2 + [(i_f - i_{\mathrm{GSO}})/w_i]^2 \tag{3}$$

In Eq. (3), h_{GSO} and i_{GSO} are the desired GSO altitude and inclination and h_f and i_f are the terminal perigee altitude and inclination of a simulation. The terms in Eq. (3) have different units. Therefore, the weight factors w_h and w_i are introduced to normalize the terms. The proper choice of the weight factors enables uniform convergence, ensuring desired error levels on both altitude and inclination. For this study, a value of 500 is used for the ratio of the weights. Other values will also work. With this value, and a convergence that achieves a fitness value of 0.999999, the error on the achieved altitude is less than 0.5 km and the error on the achieved inclination is less than 0.001 deg. The errors are different with other values.

GAAB

The initial bounds on input parameters are redefined within the existing bounds after a certain number of generations (referred to as iadapt) around the current best solution values of the parameters. The steps involved in the redefinition of the bounds are as follows.

1) Pick the best solution value b of an input parameter when the current generation is a multiple of iadapt.

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2) Compute $h_d = (h - l)/2$, where h and l are the upper and lower values of the bound, respectively.

3) If $(b - l)$ is less than h_d, then compute h_d as $(h - b)/2$. Otherwise, compute h_d as $(b - l)/2$.

4) Compute the new upper and lower values of the bound as $h = (b + h_d)$ and $l = b - h_d$.

5) Repeat steps 1–4 for all of the input parameters.

After the redefinition of the bounds, a new encoded representation of the population members must be computed with the new bounds. The solution process further proceeds with the usual steps of GA involving reproduction, crossover, and mutation.

Results and Discussion

An initial perigee altitude of 300 km and an initial inclination of 50 deg are considered for an illustrative case. A perigee altitude of 35,900 km and inclination of 1 deg are targets for the return orbit after lunar encounter. The departure is assumed in January 2007. To achieve near zero GSO inclination after lunar encounter, the declination of the moon during spacecraft's close approach must be less than the GSO inclination. From lunar ephemeris, the moon's declination is between ±1 deg on the 9th and 23rd of January 2007. The close approach must take place around these dates. In this example, we consider 23 January as the lunar encounter day, and we fix the departure epoch at 18 January, 2000, 0 hrs. The corresponding right ascension of the moon is in the range of [−3–0 deg]. The parameter Ω is varied in this region.

Precise Solution Using GAAB

The bounds were chosen as [360,000 420,000 km] for the apogee altitude and [170 190 deg] for the argument of perigee. The crossover and mutation probabilities are fixed as 0.8 and 0.01, respectively.[4] The population size is fixed at 40. For illustration, a propagation force model consisting of nonspherical gravity models of the Earth (10 × 0 field), the moon (9 × 0 field) and the sun's point mass effect is considered.

Figure 1 shows the adaptation process of GAAB. The width of the bounds, initially wide, narrows as the solution process progresses. The GA process involves selection of discrete values from the bounds for the initial parameters. In a wide bound, the values are sparsely selected and the trajectories are generated. These trajectories are highly sensitive to the initial parameters. The adaptation process enables dense discretization in the reduced bounds and overcomes the high-sensitivity problem.

The initial conditions of lunar gravity assist trajectories for a right ascension of ascending node range from −0.75 to −2.25 deg are given in Table 1. Approach flight duration ranges from 4.44 to 4.69 days. In all of the cases, the desired return orbit perigee altitude and inclination are precisely achieved. If one considers an initial transfer orbit of 300 × 35,900 km at an inclination of 50 deg, transfer to GSO by usual methods requires a velocity addition of 2350 m/s. When the lunar gravity assist as described earlier is used, the total velocity requirement would be 1767 m/s, a reduction of 583 m/s.

Effect of Earth's Oblateness

To assess the contribution of various components of the force model, GAAB initial conditions are generated using the following force models: case 1, spherical gravity fields for Earth and moon; case 2, case 1 plus Earth's second zonal harmonic; and case 3, nonspherical gravity models for Earth and moon plus point mass sun. The results are given in Table 2. These initial conditions are propagated with the case 3 force model, referred to as the reference force model. The resulting return orbits are also given in Table 2. Note the change in the initial value of apogee. The initial argument of perigee of the transfer trajectory does not change much due to force models.

Table 1 Initial conditions of precise GAAB solution

Ω, deg	h_a, km	ω, deg	Approach/total flight duration, days	Impulses at departure/ for GSO, m/s	Flyby altitude, km
−0.75	387,032.2	184.551	4.694/8.057	678.4/1090.1	8845.4
−1.0	386,724.6	184.134	4.646/7.980	678.3/1091.4	8496.7
−1.5	386,484.8	183.732	4.576/7.941	678.3/1089.3	8411.8
−2.0	386,238.2	183.148	4.493/7.844	678.2/1089.7	8047.1
−2.25	386,151.6	182.828	4.444/7.786	678.2/1090.3	7814.5

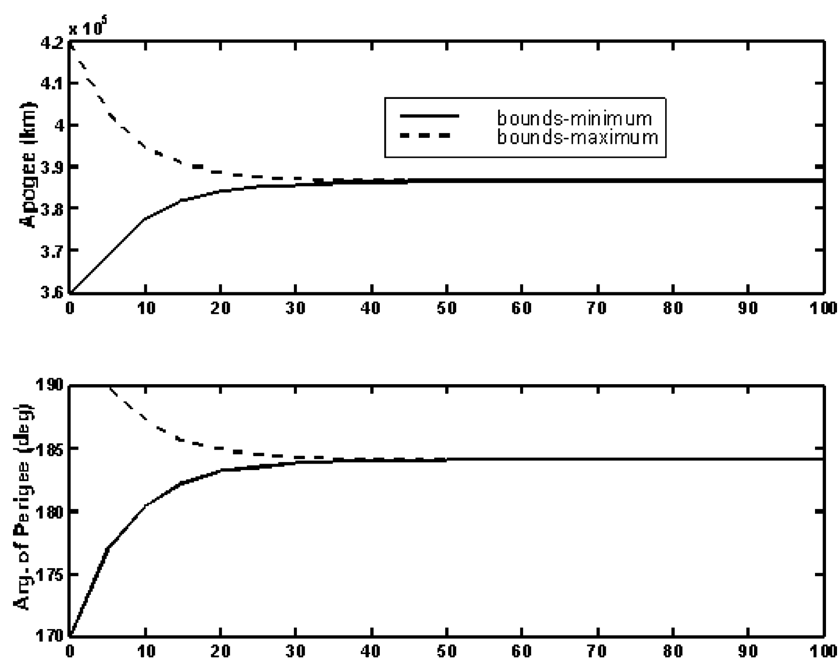

Fig. 1 Adaptation process of GAAB.

Table 2 Transfer trajectory parameters under different force models and resulting targets on propagation

Parameters	GAAB solution Case 1	Case 2	Case 3
At departure			
h_a, km	375,697.1	387,051.7	386,724.6
ω, deg	184.3360	184.2866	184.1340
Return orbit on propagation with reference force model			
Perigee altitude, km	342,259.7	33,822.2	35,900
Inclination, deg	15.54	0.44	1.0

The GAAB initial conditions of case 1 when propagated with the reference force model results in a very large return orbit perigee altitude of 342,259.7 kms. However, the GAAB solution of case 2 achieves a target perigee altitude of 33,822 kms. Furthermore, the GAAB solution of case 3 is propagated without the effect of Earth's second zonal harmonic. The spacecraft experiences a velocity change of about 2 m/s after a flight of 30 min. At this time, the spacecraft reaches a radial distance of about 14,000 kms. The velocity change due to other forces is less than 10^{-3} m/s. After this distance, the influence of asphericity is negligible. However, the initial velocity change of 2 m/s leads to large deviations in the return orbit (Table 2). It is clear that the spherical gravity models are not sufficient for precise targeting and the major contribution is from Earth's second zonal harmonic. Thus, the orbit propagation model of numerical search technique must include at least the second zonal harmonic of the Earth.

Conclusions

The design characteristics of lunar gravity assist trajectories are obtained using a numerical search technique and employing a modified version of the GA, GAAB. The operation of the search technique is demonstrated. The modified version of GA improves the convergence and helps achieve the targets' accurately. The effect of different force models used in the design process is evaluated. The second zonal harmonic of Earth's gravity plays an important role on the transfer trajectory and its achieved targets. It should be considered in the propagation force model of any numerical search process.

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